PHYS 301
Electricity and Magnetism

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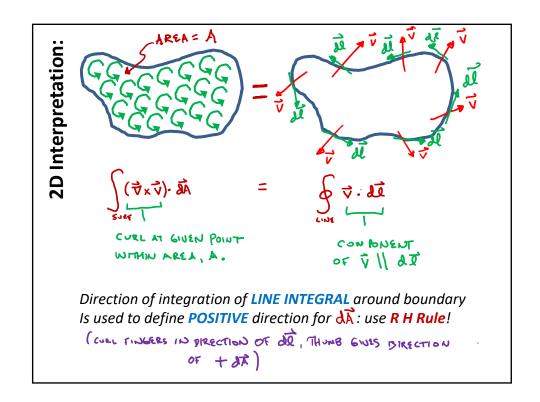
Fall 2019

Today!

- Integral vector calculus:
  - ➢ Fundamental theorem of curls
  - ➢ Fundamental theorem of divergences

Integral Calculus: Curls

- The **fundamental theorem of curls**:

$$\int_{\text{surface}} \left(\vec{\nabla} \times \vec{V} \right) \cdot d\vec{A} = \oint_{\text{line}} \vec{V} \cdot d\vec{l}$$

- Also called **STOKES' THEOREM**

- NOTE:
$$\oint \left(\vec{\nabla} \times \vec{V} \right) \cdot d\vec{A} = 0 \ \forall \text{ closed surfaces}$$
$$\int \left(\vec{\nabla} \times \vec{V} \right) \cdot d\vec{A} \text{ depends only on the boundary;}$$
e.g., soap bubble!   not on the surface!

2D Interpretation:

$$\int_{\text{surf}} (\vec{\nabla} \times \vec{v}) \cdot d\vec{A} \qquad = \qquad \oint_{\text{line}} \vec{v} \cdot d\vec{l}$$

CURL AT GIVEN POINT WITHIN AREA, A.

COMPONENT OF $\vec{v} \parallel d\vec{l}$

*Direction of integration of **LINE INTEGRAL** around boundary Is used to define **POSITIVE** direction for $d\vec{A}$: use **R H Rule**!*

(CURL FINGERS IN DIRECTION OF $d\vec{l}$, THUMB GIVES DIRECTION OF $+ d\vec{A}$)

Integral Calculus: Divergences

- The **fundamental theorem of divergences**:

$$\int_{vol} \left(\vec{\nabla} \cdot \vec{B} \right) dV = \oint_{surface} \vec{B} \cdot d\vec{A}$$

- Also called *GAUSS' THM* or *GREEN's THM*

- NOTE:
  $d\vec{A} = \hat{n}\, dA$ where $\hat{n}$ is $\perp$ the surface, pointing outward from the enclosed volume.

  $\vec{B} \cdot \hat{n} =$ component of $\vec{B} \perp$ surface.